

(Gov. Submission to
the Optical Society
of America.)

Sensibility of Radiation Detection

By Donald R. Buchele ✓

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio

N65-82129

Code-7
NASA Tex 5607
3ne

Abstract. - A performance limit for image- and radiation-detection is

derived in terms of frequency response, image resolution, and flux-density sensibility. The derivation includes the effects of statistical fluctuations of photon rate, of the limited resolving power of an image detecting system, and of the defects of a non-ideal detector. Numerical examples are given to compare the ideal detector, the photomultiplier, television camera tubes, and photographic film.

INTRODUCTION

Various analyses have been made of the performance of specific types of image- and radiation-sensitive detectors. Photographic film^{1,2,3}, television camera tubes^{1,4}, photomultipliers and other energy receptors^{2,5} have been studied.

The concepts of modern information theory⁶; and the introduction of the sampling aperture principle⁷ now make possible an alternative and more

¹A. Rose, Advances in Electronics (Academic Press, Inc., New York, 1948), Vol. I, pp. 131-166.

²R. C. Jones, Advances in Electronics (Academic Press, Inc., New York, 1953), Vol. V, pp. 1-96.

³L. Levi, J. Opt. Soc. Am. 48, 9-12, (1958).

⁴G. A. Morton and J. E. Ruedy, I.R.E. 1958 Conference Proceedings, pp. 113-117.

⁵Smith, Jones and Chasmar, The Detection and Measurement of Infra-Red Radiation (Oxford University Press, 1957).

⁶C. E. Shannon and W. Weaver, The Mathematical Theory of Communication (University of Illinois Press: Urbana, 1949).

⁷O. H. Schade, Nat'l Bur. Standards Circ. 526, 231-249 (1954).

generalized treatment of the subject than had previously been presented. It now appears possible to propose an objective definition of "threshold sensibility" and to relate it to signal-to-noise ratio.

To develop this concept, we note that the sensibility of radiant-energy detection is limited by (a) random variations in the radiant flux from signal and non-signal sources, (b) the image resolution of the detector surface or the optical system, (c) the sampling time available, and (d) internally-generated detector "noise."

The radiant flux has fluctuations due to the statistical distribution of the quantized radiation. The image resolution of an optical system is limited by the wavelength of the radiation. In photosensitive surfaces, limitations are imposed by the granularity, turbidity, and thickness of the deposited material; in electronic image-conversion devices, additional limitations are imposed by aberrations of the electronic-focusing mechanism. Detector noise appears as fog in a photographic film or as a randomly-fluctuating electrical signal from a photoelectric device.

The sampling aperture principle will be applied to image-detecting systems in order to convert image resolution to an equivalent specific detector area. By use of information theory, an equation will be developed that relates power-measurement sensibility and frequency response for various detecting systems. As in the method of Rose¹, emphasis will be placed on the statistical nature of photons, electrons, and film grains. Some calculations of threshold sensibility will be presented for inter-comparison of various detectors.

SENSIBILITY OF SIGNAL DETECTION

Information theory provides a way to relate the statistical fluctuations of photons to detector performance. Shannon⁶ has stated the maximum possible rate of transmission of a continuous waveform through a communication channel - the channel capacity. This rate can be expressed as the maximum number of binary signal units capable of being transmitted per unit time. The channel input is an average signal power P ; the channel output is P plus an average noise power N . When the channel capacity is fully utilized by the signal, both P and N must approximate, in statistical properties, a white noise with Gaussian amplitude distribution within the frequency bandwidth Δf of the channel⁶. The maximum number of bits B_{\max} of information that can be transmitted in a time duration t , as $t \rightarrow \infty$, is,

$$B_{\max} = \log_2 \left(\sqrt{\frac{P + N}{N}} \right)^{2t\Delta f} . \quad (1)$$

In this equation the quantity $2t\Delta f$ represents the smallest number of time samples that can reproduce the continuous waveform on the time axis. It is also a result of the well-known sampling theorem which states that, in the time domain, a continuous waveform t seconds in duration and band-limited in frequency to the range Δf can be completely specified by $2t\Delta f$ (

sampling points spaced $1/(2\Delta f)$ seconds apart (for $2t\Delta f \gg 1$). This gives the equation $2t\Delta f = t/\Delta t$, which introduces the sampling period Δt . Thus,

$$\Delta t = 1/(2\Delta f). \quad (2)$$

In Eq. (1) the number of bits B_{\max} may be considered as the logarithm of the number of available choices. The quantity $2t\Delta f$ exists in the time domain. The quantity $\sqrt{[(P + N)/N]}$ exists in the power domain; it represents the average number of discrete signal levels between P and $P = 0$ that can be identified without error in the presence of noise, for $t \gg \Delta t$. The average threshold power P_{th} is defined as the signal power that yields, on the average, one bit per time sample (i.e., $B_{\max}/(2t\Delta f) = 1$). Then Eq. (1) yields

$$P_{th} = 3N. \quad (3)$$

Corresponding to the rms signal power P_{rms} is an rms signal amplitude S_{rms} (current- or voltage-amplitude). Corresponding to the square root of the noise power N is a standard deviation of signal amplitude, σ_s .

Then the threshold signal amplitude is

$$S_{th} = \sqrt{3} \sigma_s. \quad (4)$$

This result is sufficient for the subsequent analysis; the number of bits transmitted is not required.

Equation (4) gives the smallest change of amplitude that can be detected in any signal, and hence constitutes the signal-amplitude sensibility. The relative signal-amplitude sensibility s_{th} is a dimensionless ratio

$$s_{th} = S_{th}/S = \sqrt{3} \sigma_s/S. \quad (5)$$

At the threshold of signal detection, $s_{th} = 1$.

The average threshold of signal detection given by Eq. (4) is less than the experimental values of $5\sigma_s$ reported by Rose¹ or $3\sigma_s$ reported by Schade⁸ for television images. It is in agreement with a value of $1.6\sigma_s$ to $2\sigma_s$ reported for the eye by Schade⁸. All of these previously published quantities represent psychophysical measurements.

STATISTICAL FLUCTUATION OF RADIATION

The performance of any radiation detector has a limit set by random fluctuations of the photon rate radiated from the source. The photon rate Q is the ratio of the photon count m to the sampling period Δt . Thus, $Q = m/\Delta t$. A precise average photon rate \bar{Q} and count \bar{m} are the averages of Q and of m for a large number of observations. Thus,

$$\bar{Q} = \frac{1}{n} \sum_{i=1}^n \frac{m_i}{\Delta t} = \frac{\bar{m}}{\Delta t} . \quad (6)$$

The measured rate Q as deduced from a small sample m has an error $Q - \bar{Q} = (m - \bar{m})/\Delta t$ due to random fluctuations of m from sample to sample. Neglecting a quantum-mechanical correction factor^{2,5}, the time intervals between successive photons are random in time and have a Poisson probability distribution. The standard deviation of the count \bar{m} is

$$\sigma_m = \sqrt{\bar{m}} \quad (7)$$

Within one period Δt the rms signal S of Eq. (5) will be considered equal to the arithmetic average photon count \bar{m} of Eq. (6), so that $S = \bar{m}$.

⁸O. H. Schade, Sr., J. Opt. Soc. Am. 46, 721-739 (1956).

Then Eqs. (2), (5), (6), and (7) combine to yield

$$s_{th} = \sqrt{(6\Delta f/\bar{Q})}. \quad (8)$$

This is the relation between relative sensibility, frequency bandwidth, and photon rate. The photon rate expressed as an average radiant flux Φ_λ at wavelength λ is

$$\Phi_\lambda = (hc/\lambda)\bar{Q} \quad (9)$$

where h is Planck's constant, c is the velocity of light.

At the threshold of signal detection where $s_{th} = 1$, Eqs. (2), (6), and (8) combine to yield $\bar{m} = 3$. Thus, three photons, on the average, are contained in one sample of the threshold signal.

Equation (8) can be written in terms of irradiancy, surface area, and exposure time, as

$$s_{th} = \sqrt{[(3hc)/(\lambda H_\lambda A \Delta t)]} \quad (10)$$

where H_λ is the average monochromatic irradiancy, Φ_λ/A , over area A for a time Δt .

IMPERFECT DETECTOR

At the output of an imperfect detector system, the noise is a result of the irreducible photon fluctuation plus other sources such as Johnson noise, shot noise, thermionic emission of a photosurface, or fog in photographic film. It may be convenient to calculate relative sensibility at the output in terms of electric current or photographic density. This may then be converted to an equivalent input using appropriate amplifier-signal-gain and detector-sensitivity factors. Eq. (5) is applicable to the imperfect detector, if σ_s therein is replaced by the square root of the weighted sum of the squares of the equivalent σ 's of each source of noise.

In studying the process of image-formation, a sampling area in the image-sensitive surface of a detector must be considered; the noise generated

by an imperfect detector usually increases with this area. Thus, the sampling area enters Eq. (5) indirectly through a σ -term for the specific detector.

The sampling area may also be used to calculate incident radiant power when the area illumination is known. In image-scanning systems, the output signal frequency response is given by the ratio of scanning velocity to diameter of the sampling area. The diameter of the sampling area is determined in the next section.

IMAGE SAMPLING AREA

From the standpoint of communication theory, an image plane can represent a multichannel communication system. Each independent area in the image represents one channel. If the areas form a mosaic surface structure, as in the retina of the eye, each channel is spatially defined by one mosaic cell. If the image plane is structureless, as in an optical image, the spatial dimension of one cell is not obvious.

This problem exists also with an unexposed photographic film. Its surface has no obvious detector element dimensions. What is needed is a criterion for the minimum size of adjacent, but effectively independent, areas on an image-sensitive surface. This criterion can be given by an effective image resolution based on the "sampling aperture" principle.⁷ The area so determined will be named the sampling area.

Optical image analysis by the sampling aperture principle begins with the fact that the image of a point is itself imperfect; assuming, for simplicity, that circular symmetry exists, a sampling aperture is obtained by replacing the relative radial intensity distribution $I(r)/I_0$ of this imperfect image point with an aperture that has a correspondingly distributed

transmission function $\tau(r)$. This is the aperture function for an optical image point.

An aperture function can also be obtained for a detector surface. If radiation impinges upon one geometric point of the detector surface, the energy will be diffused over the surface, with an attenuation that increases with the distance from the point of incidence. The resulting distribution of detector response may be considered as the aperture function of the detector surface.

Some typical round-aperture functions $\tau(r)$ are illustrated in column 1 of Table I. The exponential aperture function $\exp(-r/r_0)$ approximates that of photographic film.

In order to evaluate the quality of image formation, the aperture function is assumed to traverse a parallel-line test pattern having a sinusoidal intensity distribution with a pitch γ . The transmitted flux is sinusoidal with a pitch γ , but its amplitude is a function of γ .

In order to normalize the mathematical relations, we define first, a dimensionless variable y that is the number of wavelengths contained in a nominal aperture diameter δ (column 2 of table I), which is arbitrarily chosen as some convenient diameter representative of the aperture function $\tau(r)$; and second, a dimensionless response variable F that is the ratio of the transmitted peak-to-peak flux at abscissa y to the transmitted peak-to-peak flux as $y \rightarrow 0$. Thus

$$y = \delta/\gamma \quad \text{and} \quad F(y) = \Phi(y)/\Phi(0). \quad (11)$$

The aperture response F is shown in Fig. 1 (from Schade⁷). The condition $y \gg 1.0$ corresponds to a uniformly-illuminated image.

It is desired to compare image-forming systems with different aperture functions. This may be accomplished by arbitrarily defining a "standard"

aperture function $\tau_0(r)$ with aperture diameter δ_0 , and then determining the value of δ_0 that produces the same value of y_e as the aperture function $\tau(r)$ under consideration, where⁷

$$y_e = \int_0^\infty F^2(y) dy. \quad (12)$$

The standard aperture function chosen will be the one having a rectangular distribution (Table I), for which δ_0 is the full width of the rectangle and $y_{e,0} = 0.54$. Then

$$\delta_0 = \delta y_{e,0}/y_e = 0.54\delta/y_e = 0.54r_e. \quad (13)$$

Values of y_e are listed in Table I. The circular area $\pi\delta_0^2/4$ is the desired sampling area on an image-sensitive detector surface.

When the relation between F and $(1/r)$ is given, as in published data on television camera tubes, the value of $r_e (= 1.85\delta_0)$ can be calculated from an equation like Eq. (12).

When the "visual limit of resolution" is given (this is generally accepted as the value r_l of r where $F = 0.02$), as in published data on photographic film, the value of δ_0 is

$$\delta_0 = r_l y_{e,0} (y_l/y_e) = 0.54r_l (y_l/y_e). \quad (14)$$

Values of y_l/y_e are given in Table I. Table II lists measured values of δ_0 for photographic film and television camera tubes. A small error is present because the optical test pattern had an intensity distribution of square waveform rather than sinusoidal.

SAMPLE CALCULATION OF DETECTOR PERFORMANCE

The threshold sensibility (minimum detectable radiant signal) will be calculated in units of radiant power at 4000 Å, for the ideal detector, for a photomultiplier, and for the sampling area of television tubes and photo-

graphic film. Results are given in Fig. 2.

Threshold sensibility is equal to the signal S in Eq. (5) when $s_{th} = 1$. Thus it is first necessary to apply Eq. (5) to each detector by substituting the appropriate terms for S and σ_s . The terms may have the dimension of event rate where the event is a photon, electron, or a developed photographic film grain. Both S and σ_s will have the same frequency bandwidth.

When the threshold sensibility calculated with $s_{th} = 1$ in Eq. (5) has units of photon rate, the corresponding radiant power is obtained with Eq. (9). When the sensibility units are electron rate or film grain rate, the radiant power is obtained from a detector sensitivity graph.

Limitations on signal frequency response imposed by a detector time constant are neglected. Performance gains produced by photomultiplier refrigeration or by film pre-exposure are neglected. However, the expected performance gain over the results of Fig. 2 is a factor of two with film pre-exposure. For a photomultiplier refrigerated with liquid nitrogen, the expected gains are four at 10,000 cps and 200 at 0.01 cps.

Photomultiplier

In a photomultiplier the average signal current i_s at the load resistor is $i_s = e G \eta \bar{Q}_s$, where e is the electronic charge, G is photomultiplier gain, η is photosurface efficiency (i.e., electrons per incident photon).

The mean square noise current i_n^2 at the load resistor is the resultant of three currents:

$$i_n^2 = i_{n,p}^2 + i_{n,t}^2 + i_{n,j}^2 \quad (15)$$

where $i_{n,p}$ is due to the photon fluctuation of the incident radiation \bar{Q} , and $i_{n,t}$ is due to the photosurface thermionic emission current i_t , and $i_{n,j}$

is due to the Johnson noise voltages generated by the load resistor R_L and by an equivalent resistance R_E that represents preamplifier noise.

If \bar{Q}_b is a non-signal background photon rate, k is the Boltzmann constant, T_L is the temperature of R_L and T_E is the temperature of R_E then

$$i_{n,p}^2 = 2e^2 G^2 \eta \Delta f (\bar{Q}_s + \bar{Q}_b), \quad (16a)$$

$$i_{n,t}^2 = 2e^2 G^2 \Delta f (i_t/e), \quad (16b)$$

$$i_{n,j}^2 = (4kT_L \Delta f / R_L) [1 + (T_E R_E / T_L R_L)]. \quad (16c)$$

The frequency response is limited by the total shunting capacitance C at the load resistor. Hence R_L can be replaced by $(4C\Delta f)^{-1}$. Substitution in Eq. (5) of the above equations and setting $S = i_s$ and $\sigma_s^2 = i_n^2$, yields

$$s_{th}^2 = (6\Delta f / \eta \bar{Q}_s) [1 + (\bar{Q}/\bar{Q}_s)]. \quad (17)$$

$$\begin{aligned} \bar{Q} &= \bar{Q}_b + (i_t/e\eta) \\ &+ [8kCT_L \Delta f / (G^2 e^2 \eta)] [1 + (4T_E R_E C \Delta f / T_L)] \end{aligned}$$

The threshold sensibility, shown in Fig. 2, has been calculated for $\eta = 0.13$, $i_t = 10^{-13}$ amp (type 931-A), $\bar{Q}_b = 0$, $G = 10^6$, $T_L = T_E = 300$ K, $C = 10^{-11}$ farad, $R_E = 100$ ohms.

Television Camera Tubes

The performance of one sampling area within a standard size raster is to be found. In order to isolate the sampling area for analysis, the output of the camera tube amplifier is gated to transmit only during the time of sampling by the tube's scanning beam. The performance is obtained from the signal and noise found at the camera-tube load resistor during this sampling time.

In practice the sampling repetition rate may be limited by tube design to the standard frame rate of 30 per second. This limitation will be neglected.

Let α be the ratio of the raster area to the sampling area. This is also the ratio of sampling period Δt (duration of one frame) to the duration of a sample δt (duration of scanning beam sweep over sampling area). During the time Δt , the photocathode current accumulates in the tube target. Then, during the much shorter time δt , the target charge is removed as a pulse by the tube scanning beam. The average pulse current is α times greater than the average photocathode current taken over a time Δt . This pulse-type of waveform requires a frequency bandwidth of $\alpha \Delta f$ at the location of the load resistor and in the amplifier, where Δf is one-half the frame rate. The noise current for the gated amplifier is $i_{n,j} \alpha^{-1/2}$ where $i_{n,j}$ is given by Eq. (16c).

The image orthicon has the same signal and noise terms as the photomultiplier except that \bar{Q}_s , \bar{Q}_b , i_t , and Δf are multiplied by α , and $i_{n,j}^2$ is divided by α . The current i_t includes both the thermionic current and an equivalent current corresponding to that required to produce the scanning-beam noise. The equation for s_{th} is the same as Eq. (17) for the photomultiplier except that α appears in the term $[(1/\alpha) + (4T_E R_E C \Delta f / T_L)]$.

The threshold sensibility, shown in Fig. 2, has been calculated for the type 5820 tube, with $\eta = 0.051$, $i_t = 7.9 \times 10^{-17}$ amp for the sampling area, $\alpha = 4.0 \times 10^5$ using δ_0 of Table II, $\bar{Q}_b = 0$, $G = 1500$ (which includes a gain of 3 due to secondary emission from the target), $C = 2 \times 10^{-11}$ farad, $T_L = T_E = 300$ K, $R_E = 100$ ohms.

The Vidicon output noise is stated⁹ to be that of a temperature-limited diode with a plate current equal to that of the signal electrode current of the Vidicon. The Vidicon can be assigned the same signal and noise terms as the photomultiplier except that \bar{Q}_s , \bar{Q}_b , i_t , and Δf are multiplied by α , $i_{n,j}^2$ is divided by α , and $\eta \bar{Q}$ is replaced by a corresponding electron rate, i/e . The current i_t now represents an equivalent current corresponding to that required to cause the dark-current noise at the signal electrode; i_t will be considered equal to the dark current. Then the equation for s_{th} is the same as Eq. (17) for the photomultiplier, except that α appears in the term $[(1/\alpha) + (4T_E R_E C \Delta f / T_L)]$, $\eta \bar{Q}$ is replaced by i/e , and G is set equal to unity because no photomultiplication is provided.

The threshold current sensibility is calculated for the type 7038 tube with $i_t = 10^{-12}$ amp dark current of the sampling area, $\alpha = 2 \times 10^5$ using δ_0 of Table II, $\bar{Q}_b = 0$, $C = 10^{-11}$ farad, $T_L = T_E = 300$ K, $R_E = 100$ ohms. The sensibility in watts, shown in Fig. 2, is obtained using the tube handbook data giving amperes per lumen for the whole raster area, and a sensitivity factor of 0.0016 watts/lumen.

Photographic Film

The performance of one sampling area on Tri-X film will be found.

The average film density \bar{D} is given¹⁰ for a uniformly exposed film as

$$\bar{D} = 4a\bar{z} \log_{10} e / (\pi \delta_0^2) \quad (18)$$

⁹W. L. Hurford and R. J. Marian, R. C. A. Review 24, XIV, 372, Sept. 1954.

¹⁰J. H. Webb, J. Opt. Soc. Am. 45, 379-388 (1955).

where a is the area of an average-size grain, z is the average number of exposed grains in area $\pi\delta_0^2/4$. The formula is most accurate at low values of density.

In the sampling area on film the signal radiation produces an average number of grains \bar{z}_s ; the signal is $S = \bar{z}_s$.

In a uniformly exposed film the developed grains are distributed at random, as shown by autocorrelation measurements of microphotometer records.¹¹ Then for low densities, where Eq. (18) holds, the number of grains in any sample area may be considered to have a Poisson probability distribution. The mean square grain number fluctuation is

$$\sigma_z^2 = \bar{z}_s + \bar{z}_f + \bar{z}_b \quad (19)$$

where \bar{z}_f is the average grain number giving fog in unexposed film; \bar{z}_b is the average grain number produced by a non-signal background radiation.

Substitution for S and σ in Eq. (5) and conversion to density with Eq. (18) yields

$$s_{th}^2 = [1.7a/(\delta_0^2 \bar{D}_s)] [1 + (\bar{D}_b + \bar{D}_f)/\bar{D}_s]. \quad (20)$$

The average grain area a can be calculated from microdensitometer data and Eq. (22). To determine a , we note that, in Eq. (18), density is proportional to the number of grains so that

$$\frac{\sigma_D}{D} = \frac{\sigma_z}{z} \quad (21)$$

where σ_D is the standard deviation of density. Eqs. (18), (19), and (21) then yield

$$a = 1.76\delta_0^2 \sigma_D^2 / (\bar{D}_s + \bar{D}_f + \bar{D}_b). \quad (22)$$

¹¹H. J. Zweig, J. Opt. Soc. Am. 46, 805-820 (1956).

Grain area a can therefore be calculated from experimentally measured values of σ_D and $(\bar{D}_s + \bar{D}_f + \bar{D}_b)$, where δ_0 is the diameter of a round scanning aperture of the microdensitometer. Typical values for a using data of Ref. are: Tri-X, 24 micron²; Super XX, 4.0 micron²; Fine-grain Pan Duplicating Negative, 0.44 micron².

The threshold sensibility in units of density is calculated with $\delta_0 = 0.032$ cm based on the limit of resolution given in Table II, $a = 24$ micron², $\bar{D}_b = 0$, $\bar{D}_f = 0.10$. The resulting signal density is $\bar{D}_s = 0.046$. The film sensitometric curve at a total film density $\bar{D}_s + \bar{D}_f = 0.146$ shows an exposure $\bar{E}_s = 7.2 \times 10^{-10}$ watt-sec/cm² at 4000 Å. The corresponding signal flux within the sampling area is obtained, in terms of exposure and frequency response, by use of Eq. (2),

$$\Phi_s = \frac{1}{2} \pi \delta_0^2 \bar{E}_s \Delta f. \quad (23)$$

The sensibility shown in Fig. 2 for Tri-X film has been calculated with Eq. (23) and the above values of δ_0 and \bar{E}_s .

¹²L. A. Jones and G. C. Higgins, J. Opt. Soc. Am. 36, 203-227 (1946).

Table II. - Sampling Area Diameter

Film type	Limit of resolution, 30:1 contrast line target, optimum exposure $1/r_l, \text{ cm}^{-1}$	$\delta_0, \text{ cm}$
Panatomic X	1000	0.0021
Super XX	900	.0023
Tri-X	650	.0032
Television camera tube	$r_e, \text{ cm}$	$\delta_0, \text{ cm}$
Image Orthicon type 5820	0.0099	0.0050
Vidicon type 6326-A	.0056	.0028

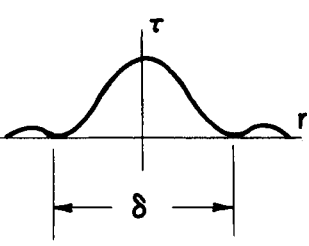
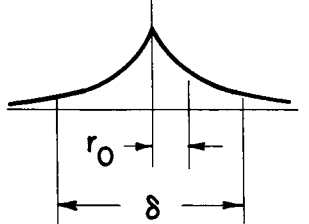
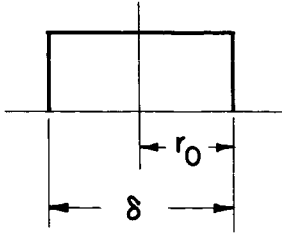
Figure Captions

Fig. 1. - Round-aperture response to parallel-bar sine-wave test pattern.

Fig. 2. - Comparative threshold sensibility for a sampling area at 4000 Å.

Table I. - Round-aperture parameters.

TABLE I

APERTURE FUNCTION	δ	y_e	y_l / y_e
 $\tau = \left[\frac{2 J_1(Z)}{Z} \right]^2$ $Z = \frac{2 \pi r (\text{N.A.})}{\lambda}$ <p>ABERRATION-FREE OPTICAL POINT IMAGE</p>	$\frac{1.22 \lambda}{\text{N.A.}}$	0.61	5.0
 $\tau = e^{-r/r_0}$ <p>EXPONENTIAL</p>	$6 r_0$.62	4.1
 $\tau = 1, r \leq r_0$ $\tau = 0, r > r_0$ <p>RECTANGULAR</p>	$2 r_0$.54	2.2

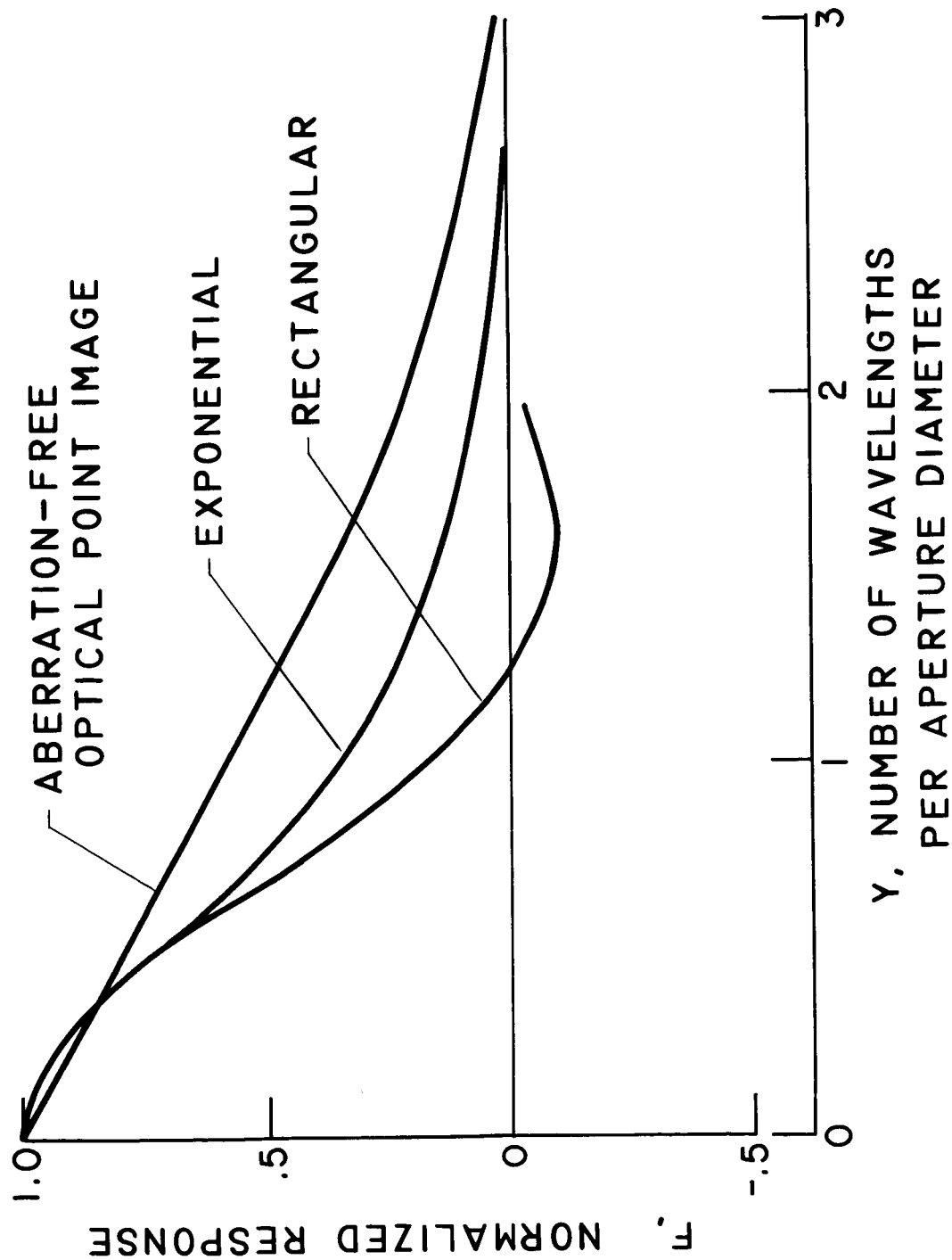


Figure 1